



# Space-time trajectories from probabilistic solar forecasts

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- What are space-time trajectories?
- Why do we need space-time trajectories?
- Space-time trajectory generation
- Space-time trajectories from probabilistic forecast
- Probabilistic and multivariate forecast assessment
- Results
- Conclusions

# What are space-time trajectories?



The most common space-time trajectories are constantly issued by meteorological institutions in the form of ensemble forecasts.

The figure shows temperature time-trajectories at a single grid point generated by the 10 ensemble members of MEPS.

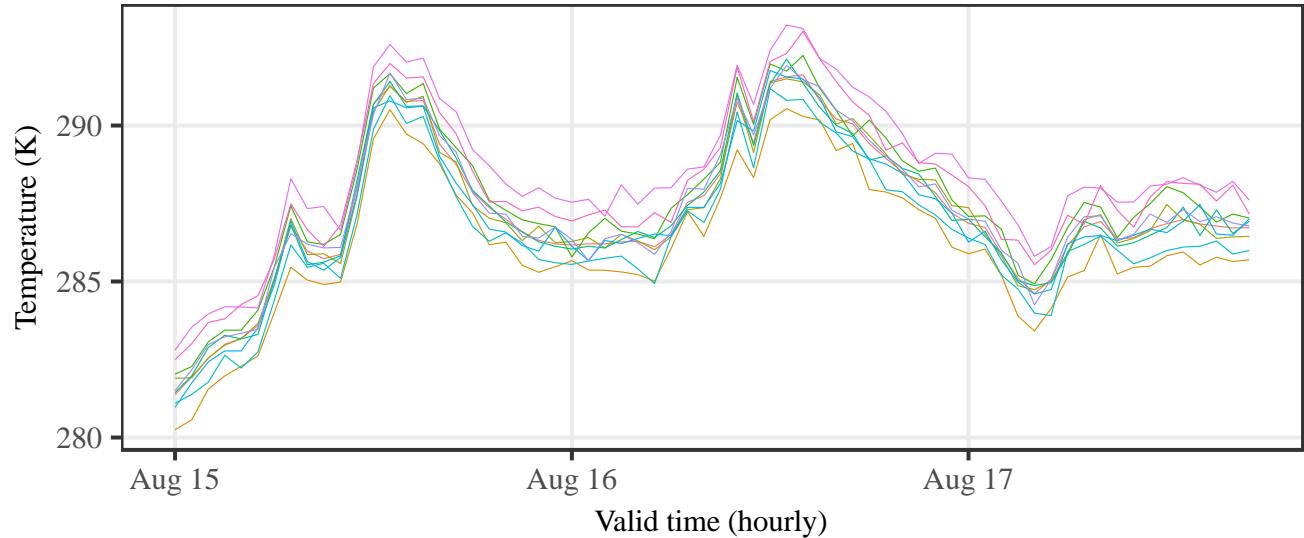


Figure: MetCoOp Ensemble Prediction System, issued 2019-08-15T00:00:00Z for Uppsala, Sweden

# Why do we need space-time trajectories?

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Because the spatio-temporal dependence structure provides essential information to operational decision problems [1], for instance:

- Electricity market participation.
- Power system reserve quantification.
- Stochastic model predictive control.
- Probabilistic power flow simulations.

# Space-time trajectories from probabilistic forecasts



- A regression or machine learning model is often used to map numerical weather prediction forecasts to e.g. PV power or to reduce systematic bias in forecasts.
- A common way to issue probabilistic forecasts is by quantile regression, such that  $q_\tau = F^{-1}(\tau)$  is the  $\tau^{\text{th}}$  quantile forecast and  $F^{-1}$  an inverse cumulative distribution function (CDF).
- By choosing e.g.  $\tau = [0.10, 0.30, \dots, 0.90]$  it is possible to issue a predictive discrete inverse CDF:

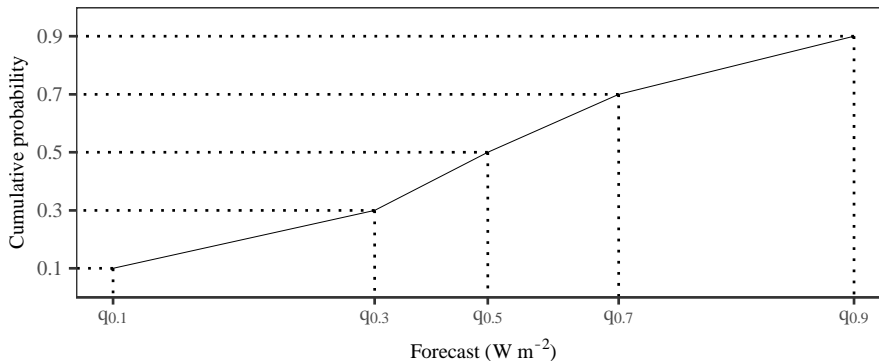


Figure: Example of a predictive discrete inverse CDF.

# Space-time trajectories from probabilistic forecasts



- Drawing a random number  $u$  from  $U[0,1]$  and interpolating, it is possible to generate a value from the discrete inverse CDF:

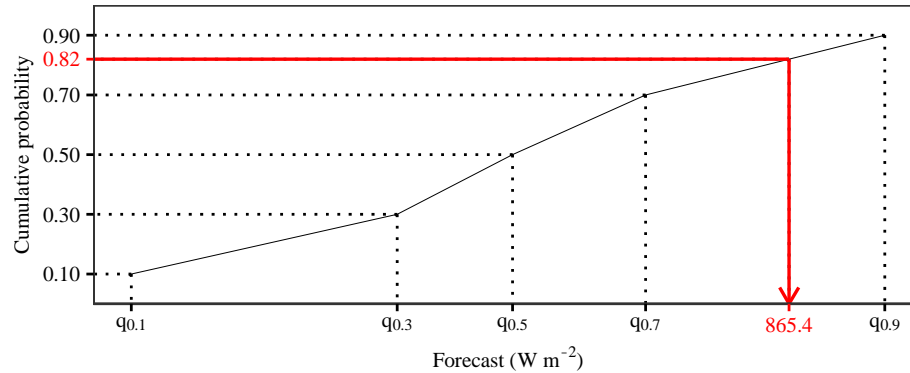


Figure: Generating a value from the discrete inverse CDF using a random number (0.82) and linear interpolation.

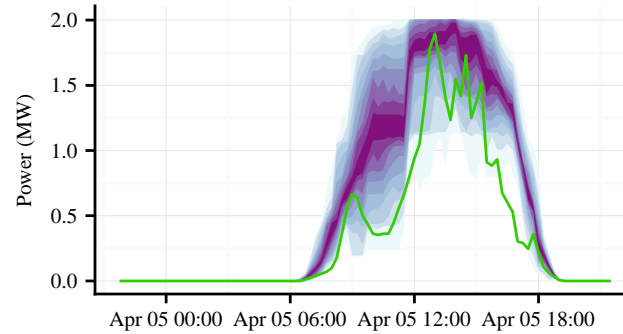
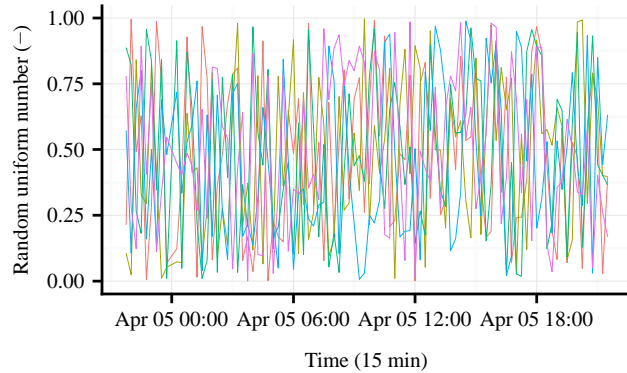
- This is known as the inverse probability integral transform:  $x = F^{-1}(u)$ .
- When probabilistic forecasts are issued for  $z = 1, \dots, Z$  locations and  $k = 1, \dots, K$  horizons,  $D = Z \times K$  random numbers are required to generate 1 space-time trajectory.

# Space-time trajectories from probabilistic forecasts



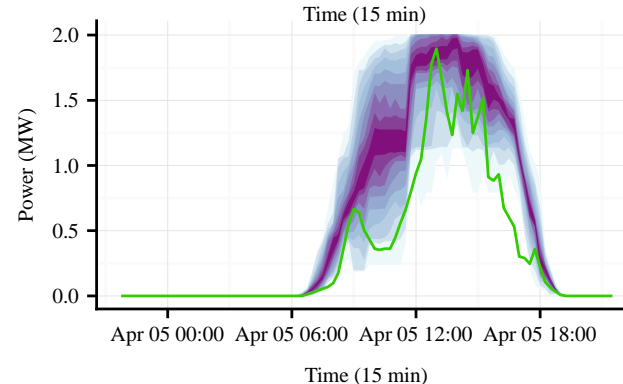
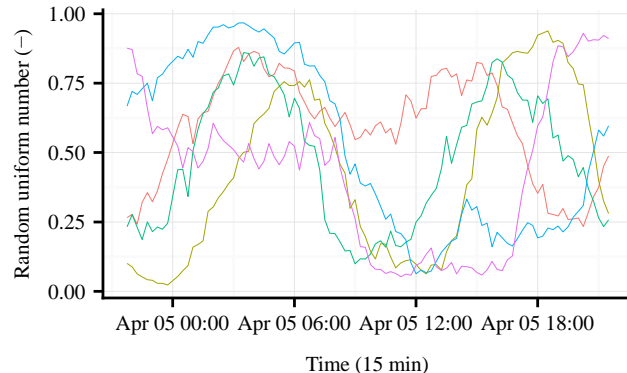
It is important that the uniform random numbers accurately represent the spatio-temporal correlation. Consider the purely temporal example ( $5 \times K$ ):

Random



PVPS

Autocorrelation



# Space-time trajectories from probabilistic forecasts



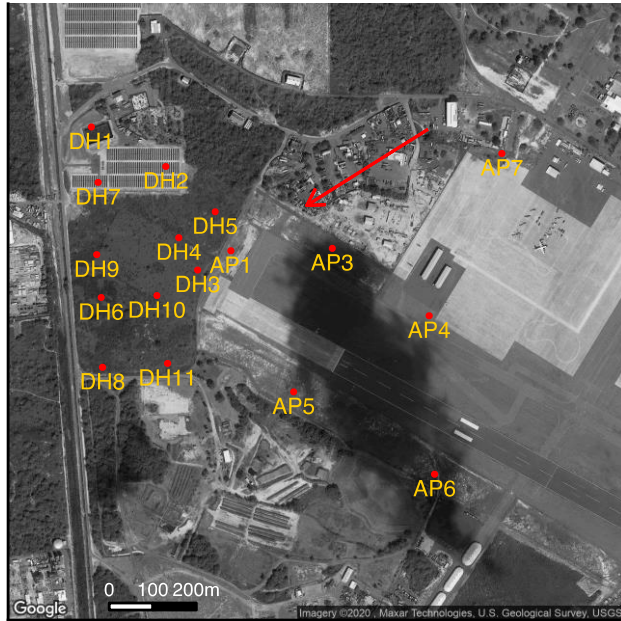
- We compare copulas to model the spatio-temporal dependence structure and sample correlated random numbers from it [2].
- Consider random variables  $(X_1, \dots, X_D)$  with CDFs  $F_{X_1}, \dots, F_{X_D}$ .
- Sklar's theorem states that every multivariate CDF can be expressed using the marginals and a copula  $C$ :  $F_{X_1, \dots, X_D}(X_1, \dots, X_D) = C(F_{X_1}(X_1), \dots, F_{X_D}(X_D))$ . [3]
- The copula can be written as:  $C(u_1, \dots, u_D) = F_{X_1, \dots, X_D}(F_{X_1}^{-1}(u_1), \dots, F_{X_D}^{-1}(u_D))$ .
- Then, it is possible to sample random uniform numbers  $(U_1, \dots, U_D)$  and generate space-time trajectories.



# Data and approach



- Aim: compare suitable copulas for multivariate solar forecasts.



PVPS

Figure: Oahu pyranometer network [4]. The red arrow indicates the prevailing wind direction. From [2].

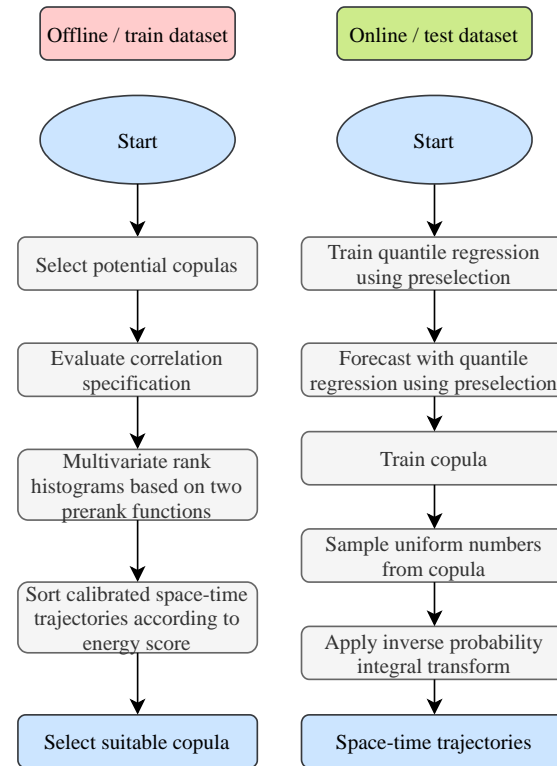


Figure: Flowchart that presents the methodology for selecting the copula (offline) and how it is used in an online setting or on the available test dataset

# Probabilistic and multivariate forecast assessment



- Probabilistic and multivariate forecasts should be calibrated for optimal decision-making processes.
- A flat rank histogram is a necessary condition for calibration, meaning that—on average—it is equiprobable for any ensemble member to predict the observation.
- Since there are  $D$  dimensions instead of 1, dedicated ‘prerank’ functions have been proposed that result in the average and band depth rank histograms [4].

$$\bullet \begin{pmatrix} y_1 & y_2 & \cdots & y_D \\ x_{1,1} & x_{1,2} & \cdots & x_{1,D} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,D} \\ \vdots & \vdots & \ddots & \vdots \\ x_{S,1} & x_{S,2} & \cdots & x_{S,D} \end{pmatrix} \text{ instead of } \begin{pmatrix} y_1 \\ x_1 \\ x_2 \\ \vdots \\ x_S \end{pmatrix}$$



- In this case, the Gaussian copula is not flexible enough to model the spatio-temporal dependence structure.
- The Student- $t$  copula results in too high correlation or underdispersed trajectories.
- The type of miscalibration from the rank histograms for the Clayton copula is inconclusive except that calibration overall is quite poor.
- The empirical copula produced slightly underdispersed trajectories caused by the probabilistic forecasts.

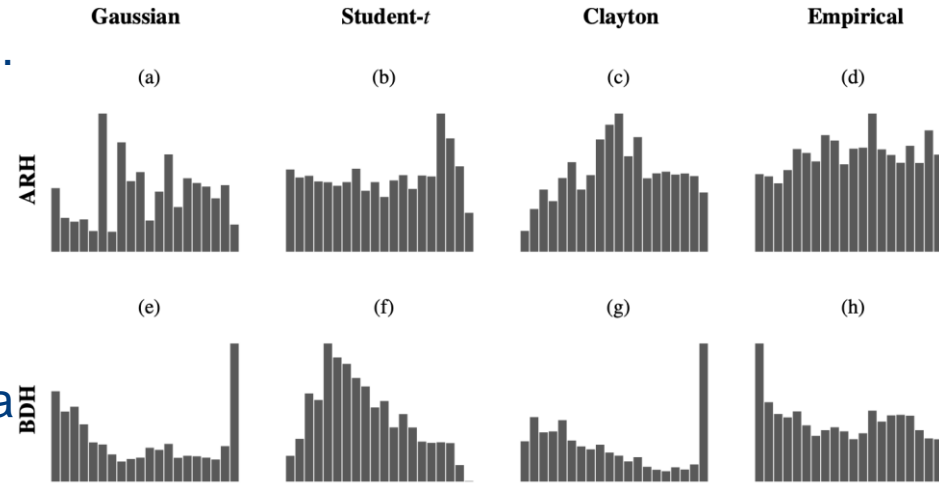


Figure: The multivariate rank histograms organized by copula (columns) and prerank function. The top row presents the average rank histogram and the bottom row represents the band depth rank histogram [2].

# Conclusions

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- Space-time trajectories are an important input to decision-making processes where the spatio-temporal relationship contains valuable information.
- Space-time trajectories can be created through several techniques but we focused on probabilistic forecasts and copulas.
- A copula is a versatile tool that allows modeling the dependence structure and marginal distributions separately.
- We found that the parametric copulas (Gaussian, Student- $t$  and Clayton) are in this case not flexible enough for the relatively large number of dimensions.
- The empirical copula showed better performance, which is probably because it is nonparametric.

**Thank you for your attention**

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# References



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- [2] D. van der Meer, D. Yang, J. Widén, and J. Munkhammar, “Clear-sky index space-time trajectories from probabilistic solar forecasts: Comparing promising copulas,” *J. Renewable Sustainable Energy* 12, 026102 (2020).
- [3] R. B. Nelsen, *An Introduction to Copulas*, Springer Series in Statistics (Springer-Verlag, Berlin, Heidelberg, 2006).
- [4] M. Sengupta and A. Andreas, “Oahu solar measurement grid (1-year archive): 1-second solar irradiance; oahu, hawaii (data),” 2010. Data retrieved from: [https://midcdmz.nrel.gov/oahu\\_archive/](https://midcdmz.nrel.gov/oahu_archive/).